Math 45 SSM 2/e 6.1 Greatest Common Factor and Factoring by Grouping (2 Days)

Objectives:

Day 1
1) Find the greatest common factor (GCF) of two or more expressions
2) Factor out the GCF in polynomials.

Day 2
3) Factor out -1 with GCF
4) Factor by grouping
   ↳ Factor out GCF 3 times in the same problem.

In chapter 5, instructions might say:

① Multiply $2x^3(3x^2-2x+1)$.

Answer: $\boxed{6x^5-4x^4+2x^3}$

Notice: We started with parentheses, and ended with no parentheses.

In chapter 6, we'll do this question backward:
We start with no parentheses, and we end with factors multiplied together using parentheses.

② Factor $6x^5-4x^4+2x^3$

Answer: $\boxed{2x^3(3x^2-2x+1)}$

step 1: How do we figure this out?

This is called finding the greatest common factor, or GCF.

$2x^3$ is the GCF of $6x^5$, $-4x^4$, and $2x^3$.

step 2: How do we figure this out?

To get this part we "factor out" the GCF.

Factor out = the opposite of multiply = we'll use division

But first, find the GCF. Only that step, by itself.

(3) Find the GCF of $6x^5$, $-4x^4$, and $2x^3$.

Notice: This question lists the terms, separated by commas. It only asks us to do step 1.

To find GCF

- Look at coefficients: 6 −4 2
  What is the largest number which divides into all the coefficients? Answer: 2
- Look at variables: x^5 x^4 x^3
  What is the largest exponent on x which can be divided from all? Answer x^3.
- Multiply these answers

$\boxed{GCF = 2x^3}$.

Shortcut: The GCF of x^5, x^4, and x^3 was x^3 → This was the smallest exponent that appeared in all three terms.

(4) Find the GCF of $12x^6y^4$, $-9x^5y^3$, $15x^7y^2$

coefficients: 12, −9, 15 ⇒ 3
variables x: x^6, x^5, x^7 ⇒ x^5
variables y: y^4, y^3, y^2 ⇒ y^2

$\boxed{GCF = 3x^5y^2}$

(5) Find the GCF of $3(x+1)^2$, $5(x+1)$, $-7(x+1)^3$

coefficients: 3, 5, −7 ⇒ 1
base (x+1) $(x+1)^2$, $(x+1)$, $(x+1)^3$ ⇒ $(x+1)$.
{treat the same as variable with a more complicated base}

$\boxed{GCF = (x+1)}$

Now, on to step 2:

(5) Factor out the GCF: (or simply) Factor

$6x^5 - 4x^4 + 2x^3$

step 1: Find the GCF (see (3))

$GCF = 2x^3$

step 2: Factor out GCF

$$2x^3\left(\frac{6x^5}{2x^3} - \frac{4x^4}{2x^3} + \frac{2x^3}{2x^3}\right)$$

numerators are the original terms

denominators are the GCF

$$= \boxed{2x^3(3x^2 - 2x + 1)}$$

Notice

$2x^3 \cdot \left(\frac{1}{2x^3}\right) = 1$

outside ↑ inside

Recall:

$\frac{6x^5}{2x^3} = \frac{6}{2} \cdot x^{5-3} = 3x^2$

* You can check a factoring problem by multiplying — you should get the original question.
(Throughout chapter 6 you can do this.)
(We did this for (6) back in problem (1)).

* Note: If the original question has 3 terms, you should have 3 terms inside the parentheses, after you factor out the GCF.

* Common Error: $\frac{2x^3}{2x^3} = 1$ not 0!

You must have the 1 in your answer.
If you don't, when you check by distributing, you'll have only 2 terms.

(7) Factor out the GCF $12x^6y^4 - 9x^5y^3 + 15x^7y^2$

step 1: Find the GCF (see (4)) $GCF = 3x^5y^2$

step 2: Factor out: $3x^5y^2\left(\frac{12x^6y^4}{3x^5y^2} - \frac{9x^5y^3}{3x^5y^2} + \frac{15x^7y^2}{3x^5y^2}\right)$ ← show this step if you need it, skip it if you don't.

$$= \boxed{3x^5y^2(4xy^2 - 3y + 5x^2)}$$

(8) Factor $\underbrace{3y^2(x+1)^2}_{\text{term \#1}} + \underbrace{5y^3(x+1)}_{\text{term \#2}} + \underbrace{7y^4(x+1)^3}_{\text{term \#3}}$

Count the terms — remember that terms are separated by + signs which are outside the parentheses. Ignore + and − signs inside parentheses.

Step 1: Find the GCF

coefficients: $3, 5, 7 \Rightarrow 1$

variable y: $y^2, y^3, y^4 \Rightarrow y^2$

base $(x+1)$: $(x+1)^2, (x+1), (x+1)^3 \Rightarrow (x+1)$

$GCF = y^2(x+1)$.

Step 2: Factor out GCF.

$$y^2(x+1)\left[\frac{3y^2(x+1)^2}{y^2(x+1)} + \frac{5y^3(x+1)}{y^2(x+1)} + \frac{7y^4(x+1)^3}{y^2(x+1)}\right]$$

$$= \boxed{y^2(x+1)\left[3(x+1) + 5y + 7y^2(x+1)^2\right]}$$

Note: Later in chapter 6, we'll do more with the bracketed part. But for now we only know the GCF.

* Instructions in chapter 6 are usually "Factor" or "Factor completely".

⇒ We will learn at least one and sometimes several factoring methods in each section of chapter 6. The instructions will say the same thing, so we have to know when to use which method.

(8½) $12x^4y^6z - 18x^5y^5z + 24x^3y^4z^3 - 6x^3y^4z)$

$$= \boxed{6x^3y^4z(2xy^2z - 3x^2y + 4z^2 - 1)}$$

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Factor

(9) $3(x-1) + 8x(x-1)$

$= \boxed{(x-1)(3+8x)}$

GCF

(10) $2(z+3)(z+5) + 4(z+5)^2$

$= 2(z+5)\left[(z+3) + 2(z+5)\right]$ ← The bracketed part can be simplified.

GCF; left from 1st term; left from 2nd term

$= 2(z+5)(z+3+2z+10)$

$= \boxed{2(z+5)(3z+13)}$

(11) $(x-2)(x-3) + (x-2)$

$= (x-2)\left[x-3 + 1\right]$

GCF; 1st; 2nd

$= (x-2)(x-2)$

$= \boxed{(x-2)^2}$

(12) $x^2(4x+1) + 2x(4x+1) + 5(4x+1)$

$= \boxed{(4x+1)(x^2+2x+5)}$

GCF; left 1st term; left 2nd term; left 3rd term